

Pensieve header: The \$k=2\$ building blocks.

```

In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
<< SL2Invariant.m
$QZipFail = True;
Block[{$k = 2}, atoms = {
  am -> am_{i,j->k}, bm -> bm_{i,j->k}, dm -> dm_{i,j->k}, R -> R_{i,j}, R -> R_{i,j}, P -> P_{i,j},
  aS -> aS_i, aS -> aS_i, bS -> bS_i, bS -> bS_i, dS -> dS_i, aD -> aD_{i->j,k}, bD -> bD_{i->j,k},
  dD -> dD_{i->j,k}, C -> C_i, C -> C_i, Kink -> Kink_i, Kink -> Kink_i, b2t -> b2t_i, t2b -> t2b_i
}] //
Column
    
```

Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: June 2018. Original version: July 1994.

$$am \rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[a_k (\alpha_i + \alpha_j), x_k (e^{-\gamma \alpha_j} \xi_i + \xi_j), 1 \right]$$

$$bm \rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[b_k (\beta_i + \beta_j), y_k (\eta_i + \eta_j), 1 - y_k \beta_i \eta_j \in + \frac{1}{2} (y_k \beta_i^2 \eta_j + y_k^2 \beta_i^2 \eta_j^2) \epsilon^2 + O[\epsilon^3] \right]$$

$$dm \rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[a_k \alpha_i + a_k \alpha_j + b_k \beta_i + b_k \beta_j, y_k \eta_i + \frac{y_k \eta_j}{\mathcal{A}_i} + \frac{x_k \xi_i}{\mathcal{A}_j} + \frac{(1-B_k) \eta_j \xi_i}{\hbar} + x_k \xi_j, \right.$$

$$\begin{aligned}
 & 1 + \left(-\frac{y_k \beta_i \eta_j}{\mathcal{A}_i} - \frac{x_k \beta_j \xi_i}{\mathcal{A}_j} + a_k B_k \eta_j \xi_i + \frac{\gamma \hbar x_k y_k \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} + \frac{(\gamma-3 \gamma B_k) y_k \eta_j^2 \xi_i}{2 \mathcal{A}_i} + \frac{(\gamma-3 \gamma B_k) x_k \eta_j \xi_i^2}{2 \mathcal{A}_j} + \frac{(\gamma-4 \gamma B_k + 3 \gamma B_k^2) \eta_j^2 \xi_i^2}{4 \hbar} \right) \epsilon + \\
 & \left(\frac{y_k \beta_i^2 \eta_j}{2 \mathcal{A}_i} + \frac{y_k^2 \beta_j^2 \eta_j^2}{2 \mathcal{A}_j^2} + \frac{x_k \beta_j^2 \xi_i}{2 \mathcal{A}_j} - \frac{1}{2} \hbar a_k^2 B_k \eta_j \xi_i + \frac{\gamma^2 \hbar^2 x_k y_k \eta_j \xi_i}{2 \mathcal{A}_i \mathcal{A}_j} - \frac{\gamma \hbar x_k y_k \beta_i \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} - \frac{\gamma \hbar x_k y_k \beta_j \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} + \frac{x_k y_k \beta_i \beta_j \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} + \right. \\
 & \frac{3 \gamma \hbar a_k B_k y_k \eta_j^2 \xi_i}{2 \mathcal{A}_i} + \frac{(\gamma^2 \hbar - 5 \gamma^2 \hbar B_k) y_k \eta_j^2 \xi_i}{4 \mathcal{A}_i} + \frac{\gamma^2 \hbar^2 x_k y_k^2 \eta_j^2 \xi_i}{2 \mathcal{A}_i^2 \mathcal{A}_j} - \frac{a_k B_k y_k \beta_i \eta_j^2 \xi_i}{\mathcal{A}_i} + \frac{(-\gamma + 3 \gamma B_k) y_k \beta_i \eta_j^2 \xi_i}{2 \mathcal{A}_i} - \frac{\gamma \hbar x_k y_k^2 \beta_i \eta_j^2 \xi_i}{\mathcal{A}_i^2 \mathcal{A}_j} + \\
 & \frac{(\gamma^2 \hbar - 7 \gamma^2 \hbar B_k) y_k^2 \eta_j^2 \xi_i}{6 \mathcal{A}_i^2} + \frac{(-\gamma + 3 \gamma B_k) y_k^2 \beta_i \eta_j^2 \xi_i}{2 \mathcal{A}_i^2} + \frac{x_k^2 \beta_j^2 \xi_i^2}{2 \mathcal{A}_j^2} + \frac{\gamma^2 \hbar^2 x_k^2 y_k \eta_j \xi_i^2}{2 \mathcal{A}_i^2 \mathcal{A}_j^2} + \frac{3 \gamma \hbar a_k B_k x_k \eta_j \xi_i^2}{2 \mathcal{A}_i} + \frac{(\gamma^2 \hbar - 5 \gamma^2 \hbar B_k) x_k \eta_j \xi_i^2}{4 \mathcal{A}_j} - \\
 & \frac{\gamma \hbar x_k^2 y_k \beta_j \eta_j \xi_i^2}{\mathcal{A}_i \mathcal{A}_j^2} - \frac{a_k B_k x_k \beta_j \eta_j \xi_i^2}{\mathcal{A}_j} + \frac{(-\gamma + 3 \gamma B_k) x_k \beta_j \eta_j \xi_i^2}{2 \mathcal{A}_j} + \frac{1}{2} a_k^2 B_k^2 \eta_j^2 \xi_i^2 + \frac{1}{2} a_k (2 \gamma B_k - 3 \gamma B_k^2) \eta_j^2 \xi_i^2 + \\
 & \frac{1}{8} (\gamma^2 - 6 \gamma^2 B_k + 5 \gamma^2 B_k^2) \eta_j^2 \xi_i^2 + \frac{\gamma^2 \hbar^2 x_k^2 y_k^2 \eta_j^2 \xi_i^2}{2 \mathcal{A}_i^2 \mathcal{A}_j^2} + \frac{\gamma \hbar a_k B_k x_k y_k \eta_j^2 \xi_i^2}{\mathcal{A}_i \mathcal{A}_j} + \frac{(5 \gamma^2 \hbar - 21 \gamma^2 \hbar B_k) x_k y_k \eta_j^2 \xi_i^2}{4 \mathcal{A}_i \mathcal{A}_j} + \\
 & \frac{(-\gamma + 3 \gamma B_k) x_k y_k \beta_i \eta_j^2 \xi_i^2}{2 \mathcal{A}_i \mathcal{A}_j} + \frac{(-\gamma + 3 \gamma B_k) x_k y_k \beta_j \eta_j^2 \xi_i^2}{2 \mathcal{A}_i \mathcal{A}_j} + \frac{a_k (\gamma B_k - 3 \gamma B_k^2) y_k \eta_j^2 \xi_i^2}{2 \mathcal{A}_i} + \frac{(5 \gamma^2 - 34 \gamma^2 B_k + 41 \gamma^2 B_k^2) y_k \eta_j^2 \xi_i^2}{12 \mathcal{A}_i} + \\
 & \frac{(\gamma^2 \hbar - 3 \gamma^2 \hbar B_k) x_k y_k^2 \eta_j^2 \xi_i^2}{2 \mathcal{A}_i^2 \mathcal{A}_j} + \frac{(-\gamma + 4 \gamma B_k - 3 \gamma B_k^2) y_k \beta_i \eta_j^2 \xi_i^2}{4 \hbar \mathcal{A}_i} + \frac{(\gamma^2 - 6 \gamma^2 B_k + 9 \gamma^2 B_k^2) y_k^2 \eta_j^4 \xi_i^2}{8 \mathcal{A}_i^2} + \frac{(\gamma^2 \hbar - 7 \gamma^2 \hbar B_k) x_k^2 \eta_j \xi_i^2}{6 \mathcal{A}_j^2} + \\
 & \frac{(-\gamma + 3 \gamma B_k) x_k^2 \beta_j \eta_j \xi_i^2}{2 \mathcal{A}_j^2} + \frac{(\gamma^2 \hbar - 3 \gamma^2 \hbar B_k) x_k^2 y_k \eta_j^2 \xi_i^2}{2 \mathcal{A}_i \mathcal{A}_j^2} + \frac{a_k (\gamma B_k - 3 \gamma B_k^2) x_k \eta_j^2 \xi_i^2}{2 \mathcal{A}_j} + \frac{(5 \gamma^2 - 34 \gamma^2 B_k + 41 \gamma^2 B_k^2) x_k \eta_j^2 \xi_i^2}{12 \mathcal{A}_j} + \\
 & \frac{(-\gamma + 4 \gamma B_k - 3 \gamma B_k^2) x_k \beta_j \eta_j^2 \xi_i^2}{4 \hbar \mathcal{A}_j} + \frac{a_k (\gamma B_k - 4 \gamma B_k^2 + 3 \gamma B_k^3) \eta_j^2 \xi_i^2}{4 \hbar} + \frac{(5 \gamma^2 - 39 \gamma^2 B_k + 75 \gamma^2 B_k^2 - 41 \gamma^2 B_k^3) \eta_j^2 \xi_i^2}{36 \hbar} + \\
 & \frac{(\gamma^2 - 5 \gamma^2 B_k + 6 \gamma^2 B_k^2) x_k y_k \eta_j^2 \xi_i^2}{2 \mathcal{A}_i \mathcal{A}_j} + \frac{(\gamma^2 - 7 \gamma^2 B_k + 15 \gamma^2 B_k^2 - 9 \gamma^2 B_k^3) y_k \eta_j^4 \xi_i^2}{8 \hbar \mathcal{A}_i} + \frac{(\gamma^2 - 6 \gamma^2 B_k + 9 \gamma^2 B_k^2) x_k^2 \eta_j^2 \xi_i^2}{8 \mathcal{A}_j^2} + \\
 & \left. \frac{(\gamma^2 - 7 \gamma^2 B_k + 15 \gamma^2 B_k^2 - 9 \gamma^2 B_k^3) x_k \eta_j^2 \xi_i^2}{8 \hbar \mathcal{A}_j} + \frac{(\gamma^2 - 8 \gamma^2 B_k + 22 \gamma^2 B_k^2 - 24 \gamma^2 B_k^3 + 9 \gamma^2 B_k^4) \eta_j^4 \xi_i^2}{32 \hbar^2} \right) \epsilon^2 + O[\epsilon^3]
 \end{aligned}$$

$$R \rightarrow \mathbb{E}_{\{i\} \rightarrow \{i,j\}} \left[\hbar a_j b_i, \hbar x_j y_i, 1 - \frac{1}{4} (\gamma \hbar^3 x_j^2 y_i^2) \epsilon + \left(\frac{1}{9} \gamma^2 \hbar^5 x_j^3 y_i^3 + \frac{1}{32} \gamma^2 \hbar^6 x_j^4 y_i^4 \right) \epsilon^2 + O[\epsilon^3] \right]$$

$$\bar{R} \rightarrow \mathbb{E}_{\{i\} \rightarrow \{i,j\}} \left[-\hbar a_j b_i, -\frac{\hbar x_j y_i}{B_i}, 1 + \left(-\frac{\hbar^2 a_j x_j y_i}{B_i} - \frac{3 \gamma \hbar^3 x_j^2 y_i^2}{4 B_i^2} \right) \epsilon + \right.$$

$$\left. \left(-\frac{\hbar^3 a_j^2 x_j y_i}{2 B_i} + \frac{\gamma^2 \hbar^4 x_j^3 y_i^2}{2 B_i^2} - \frac{3 \gamma \hbar^4 a_j x_j^2 y_i^2}{2 B_i^2} + \frac{\hbar^4 a_j^2 x_j^2 y_i^2}{2 B_i^2} - \frac{10 \gamma^2 \hbar^5 x_j^3 y_i^3}{9 B_i^2} + \frac{3 \gamma \hbar^5 a_j x_j^3 y_i^3}{4 B_i^2} + \frac{9 \gamma^2 \hbar^6 x_j^4 y_i^4}{32 B_i^2} \right) \epsilon^2 + O[\epsilon^3] \right]$$

$$P \rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{i\}} \left[\frac{\alpha_j \beta_i}{\hbar}, \frac{\eta_i \xi_j}{\hbar}, 1 + \frac{\gamma \eta_j^2 \xi_j^2 \epsilon}{4 \hbar} + \left(\frac{1}{8} \gamma^2 \eta_i^2 \xi_j^2 + \frac{\gamma^2 \eta_j^2 \xi_j^2}{4 \hbar} + \frac{\gamma^2 \eta_i^4 \xi_j^4}{16 \hbar^2} - \frac{\gamma^2 (32 \hbar \eta_j^3 \xi_j^3 + 9 \eta_i^4 \xi_j^4)}{288 \hbar^2} \right) \epsilon^2 + O[\epsilon^3] \right]$$

$$\begin{aligned}
\mathbf{aS} &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[-\mathbf{a}_i \alpha_i, -\mathbf{x}_i \mathcal{A}_i \xi_i, \right. \\
&\quad \mathbf{1} + \left(-\hbar \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma \hbar \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 \right) \epsilon + \left(-\frac{1}{2} \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \right. \\
&\quad \left. \frac{1}{2} \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2} \gamma^2 \hbar^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{1}{2} \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 \right) \epsilon^2 + \mathbf{O}[\epsilon]^3 \Big] \\
\overline{\mathbf{aS}} &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[-\mathbf{a}_i \alpha_i, -\mathbf{x}_i \mathcal{A}_i \xi_i, \mathbf{1} + \left(\gamma \hbar \mathbf{x}_i \mathcal{A}_i \xi_i - \hbar \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma \hbar \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 \right) \epsilon + \right. \\
&\quad \left(-\gamma^2 \hbar^2 \mathbf{x}_i \mathcal{A}_i \xi_i + 2 \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{7}{2} \gamma^2 \hbar^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \right. \\
&\quad \left. 4 \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - 2 \gamma^2 \hbar^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 + \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 + \right. \\
&\quad \left. \frac{1}{8} \left(4 \gamma^2 \hbar^2 \mathbf{x}_i \mathcal{A}_i \xi_i - 8 \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i + 4 \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i - 14 \gamma^2 \hbar^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + 16 \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \right. \right. \\
&\quad \left. \left. 4 \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + 8 \gamma^2 \hbar^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - 4 \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \gamma^2 \hbar^2 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 \right) \right) \epsilon^2 + \mathbf{O}[\epsilon]^3 \Big] \\
\mathbf{bS} &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[-\mathbf{b}_i \beta_i, -\frac{\mathbf{y}_i \eta_i}{\mathbf{B}_i}, \mathbf{1} + \left(-\frac{\mathbf{y}_i \beta_i \eta_i}{\mathbf{B}_i} - \frac{\gamma \hbar \mathbf{y}_i^2 \eta_i^2}{2 \mathbf{B}_i^2} \right) \epsilon + \right. \\
&\quad \left(-\frac{\mathbf{y}_i \beta_i^2 \eta_i}{2 \mathbf{B}_i} + \frac{\gamma^2 \hbar^2 \mathbf{y}_i^2 \eta_i^2}{4 \mathbf{B}_i^2} - \frac{\gamma \hbar \mathbf{y}_i^2 \beta_i \eta_i^2}{\mathbf{B}_i^2} + \frac{\mathbf{y}_i^2 \beta_i^2 \eta_i^2}{2 \mathbf{B}_i^2} - \frac{\gamma^2 \hbar^2 \mathbf{y}_i^3 \eta_i^3}{2 \mathbf{B}_i^3} + \frac{\gamma \hbar \mathbf{y}_i^3 \beta_i \eta_i^3}{2 \mathbf{B}_i^2} + \frac{\gamma^2 \hbar^2 \mathbf{y}_i^4 \eta_i^4}{8 \mathbf{B}_i^4} \right) \epsilon^2 + \mathbf{O}[\epsilon]^3 \Big] \\
\overline{\mathbf{bS}} &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[-\mathbf{b}_i \beta_i, -\frac{\mathbf{y}_i \eta_i}{\mathbf{B}_i}, \mathbf{1} + \left(\frac{\gamma \hbar \mathbf{y}_i \eta_i}{\mathbf{B}_i} - \frac{\mathbf{y}_i \beta_i \eta_i}{\mathbf{B}_i} - \frac{\gamma \hbar \mathbf{y}_i^2 \eta_i^2}{2 \mathbf{B}_i^2} \right) \epsilon + \right. \\
&\quad \left(-\frac{\gamma^2 \hbar^2 \mathbf{y}_i \eta_i}{2 \mathbf{B}_i} + \frac{\gamma \hbar \mathbf{y}_i \beta_i \eta_i}{\mathbf{B}_i} - \frac{\mathbf{y}_i \beta_i^2 \eta_i}{2 \mathbf{B}_i} + \frac{7 \gamma^2 \hbar^2 \mathbf{y}_i^2 \eta_i^2}{4 \mathbf{B}_i^2} - \frac{2 \gamma \hbar \mathbf{y}_i^2 \beta_i \eta_i^2}{\mathbf{B}_i^2} + \frac{\mathbf{y}_i^2 \beta_i^2 \eta_i^2}{2 \mathbf{B}_i^2} - \frac{\gamma^2 \hbar^2 \mathbf{y}_i^3 \eta_i^3}{\mathbf{B}_i^3} + \frac{\gamma \hbar \mathbf{y}_i^3 \beta_i \eta_i^3}{2 \mathbf{B}_i^2} + \frac{\gamma^2 \hbar^2 \mathbf{y}_i^4 \eta_i^4}{8 \mathbf{B}_i^4} \right) \epsilon^2 + \mathbf{O}[\epsilon]^3 \Big]
\end{aligned}$$

$$\begin{aligned}
 dS \rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} & \left[-\mathbf{a}_i \alpha_i - \mathbf{b}_i \beta_i, -\frac{y_i \mathcal{A}_i \eta_i}{B_i} - \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \eta_i \xi_i}{h B_i}, \right. \\
 1 + & \left(\frac{\gamma h y_i \mathcal{A}_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{\gamma h y_i^2 \mathcal{A}_i^2 \eta_i^2}{2 B_i^2} - h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i + \frac{\mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} - \right. \\
 & \frac{\gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{h B_i} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{2 B_i^2} - \\
 & \left. \frac{1}{2} \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 h B_i^2} \right) \in + \\
 \left(-\frac{\gamma^2 h^2 y_i \mathcal{A}_i \eta_i}{2 B_i} + \frac{\gamma h y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i^2 \eta_i}{2 B_i} + \frac{7 \gamma^2 h^2 y_i^2 \mathcal{A}_i^2 \eta_i^2}{4 B_i^2} - \frac{2 \gamma h y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{B_i^2} + \frac{y_i^2 \mathcal{A}_i^2 \beta_i^2 \eta_i^2}{2 B_i^2} - \frac{\gamma^2 h^2 y_i^3 \mathcal{A}_i^3 \eta_i^3}{B_i^2} + \right. \\
 & \frac{\gamma h y_i^3 \mathcal{A}_i^3 \beta_i \eta_i^3}{2 B_i^2} + \frac{\gamma^2 h^2 y_i^4 \mathcal{A}_i^4 \eta_i^4}{8 B_i^4} - \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i - h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i - \frac{1}{2} \mathbf{x}_i \mathcal{A}_i \beta_i^2 \xi_i - \frac{\gamma h \mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} + \\
 \text{Out[*]} = & \frac{h \mathbf{a}_i^2 \mathcal{A}_i \eta_i \xi_i}{2 B_i} + \frac{3 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{2 B_i} - \frac{2 \gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \frac{(\gamma^2 h \mathcal{A}_i - \gamma^2 h B_i \mathcal{A}_i) \eta_i \xi_i}{2 B_i} + \frac{\mathbf{a}_i \mathcal{A}_i \beta_i \eta_i \xi_i}{B_i} - \\
 & \frac{3 \gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{h \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{B_i} + \frac{\mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i^2 \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{2 h B_i} + \\
 & \frac{5 \gamma h \mathbf{a}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} - \frac{5 \gamma^2 h^2 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} + \frac{\gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (-21 \gamma^2 h \mathcal{A}_i^2 + 9 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{4 B_i^2} - \frac{\mathbf{a}_i y_i \mathcal{A}_i^2 \beta_i \eta_i^2 \xi_i}{B_i^2} + \\
 & \frac{3 \gamma h \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (5 \gamma \mathcal{A}_i^2 - 3 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i}{B_i^2} + \frac{y_i (-\mathcal{A}_i^2 + B_i \mathcal{A}_i^2) \beta_i^2 \eta_i^2 \xi_i}{h B_i^2} - \frac{\gamma h \mathbf{a}_i y_i^2 \mathcal{A}_i^2 \eta_i^3 \xi_i}{2 B_i^2} + \frac{\gamma^2 h^2 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^3 \xi_i}{2 B_i^2} + \\
 & \frac{y_i^2 (14 \gamma^2 h \mathcal{A}_i^2 - 5 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^3 \xi_i}{3 B_i^2} + \frac{y_i^2 (-2 \gamma \mathcal{A}_i^2 + \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^3 \xi_i}{B_i^2} + \frac{y_i^2 (-3 \gamma^2 h \mathcal{A}_i^2 + \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^4 \xi_i}{4 B_i^4} + \frac{1}{4} \gamma^2 h^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \\
 & \gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 + h \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 + \frac{1}{2} \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i^2 \xi_i^2 - \frac{h \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \eta_i \xi_i^2}{B_i} - \\
 & \frac{2 \gamma^2 h^2 \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{B_i} + \frac{\gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{B_i} + \frac{\mathbf{a}_i \mathbf{x}_i (8 \gamma h \mathcal{A}_i^2 - 3 \gamma h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{x}_i (-11 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{4 B_i} + \\
 & \frac{3 \gamma h \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{a}_i \mathbf{x}_i (-2 \mathcal{A}_i^2 + B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{x}_i (4 \gamma \mathcal{A}_i^2 - 2 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{x}_i (-\mathcal{A}_i^2 + B_i \mathcal{A}_i^2) \beta_i^2 \eta_i \xi_i^2}{h B_i} + \\
 & \frac{\mathbf{a}_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{3 \gamma^2 h^2 \mathbf{x}_i^2 y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{\mathbf{a}_i (-5 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{(21 \gamma^2 \mathcal{A}_i^2 - 30 \gamma^2 B_i \mathcal{A}_i^2 + 9 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{8 B_i^2} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i y_i (-5 \gamma h \mathcal{A}_i^2 + \gamma h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{\mathbf{x}_i y_i (31 \gamma^2 h \mathcal{A}_i^2 - 11 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{\mathbf{a}_i (\mathcal{A}_i^2 - B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{h B_i^2} + \\
 & \frac{(-5 \gamma \mathcal{A}_i^2 + 8 \gamma B_i \mathcal{A}_i^2 - 3 \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{2 h B_i^2} + \frac{\mathbf{x}_i y_i (-4 \gamma \mathcal{A}_i^2 + 2 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{B_i^2} + \frac{(\mathcal{A}_i^2 - 2 B_i \mathcal{A}_i^2 + B_i^2 \mathcal{A}_i^2) \beta_i^2 \eta_i^2 \xi_i^2}{2 h^2 B_i^2} + \\
 & \frac{\mathbf{a}_i y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{2 B_i^2} + \frac{y_i (-34 \gamma^2 \mathcal{A}_i^2 + 35 \gamma^2 B_i \mathcal{A}_i^2 - 7 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{6 B_i^2} + \frac{\mathbf{x}_i y_i^2 (-9 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{4 B_i^2} + \\
 & \frac{y_i (9 \gamma \mathcal{A}_i^2 - 12 \gamma B_i \mathcal{A}_i^2 + 3 \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^3 \xi_i^2}{4 h B_i^2} + \frac{y_i^2 (6 \gamma^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i \mathcal{A}_i^2 + \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^4 \xi_i^2}{4 B_i^4} - \frac{1}{2} \gamma^2 h^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^3 + \\
 & \frac{1}{2} \gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^3 + \frac{1}{2} \gamma h \mathbf{x}_i^3 \mathcal{A}_i^2 \beta_i \xi_i^3 + \frac{\gamma^2 h^2 \mathbf{x}_i^3 y_i \mathcal{A}_i^2 \eta_i \xi_i^3}{2 B_i} + \frac{\mathbf{a}_i \mathbf{x}_i^2 (-4 \gamma h \mathcal{A}_i^2 + \gamma h B_i \mathcal{A}_i^2) \eta_i \xi_i^3}{2 B_i} + \\
 & \frac{\mathbf{x}_i^2 (19 \gamma^2 h \mathcal{A}_i^2 - 7 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^3}{6 B_i} + \frac{\mathbf{x}_i^2 (-2 \gamma \mathcal{A}_i^2 + \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^3}{B_i} + \frac{\mathbf{a}_i \mathbf{x}_i (9 \gamma \mathcal{A}_i^2 - 6 \gamma B_i \mathcal{A}_i^2 + \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{4 B_i^2} + \\
 & \frac{\mathbf{x}_i (-59 \gamma^2 \mathcal{A}_i^2 + 58 \gamma^2 B_i \mathcal{A}_i^2 - 11 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{12 B_i^2} + \frac{\mathbf{x}_i^2 y_i (-9 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{4 B_i^2} + \frac{\mathbf{x}_i (9 \gamma \mathcal{A}_i^2 - 12 \gamma B_i \mathcal{A}_i^2 + 3 \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^3}{4 h B_i^2} + \\
 & \frac{\mathbf{a}_i (-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \eta_i^3 \xi_i^3}{4 h B_i^2} + \frac{(34 \gamma^2 \mathcal{A}_i^2 - 69 \gamma^2 B_i \mathcal{A}_i^2 + 42 \gamma^2 B_i^2 \mathcal{A}_i^2 - 7 \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^3 \xi_i^3}{18 h B_i^2} + \frac{\mathbf{x}_i y_i (6 \gamma^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i \mathcal{A}_i^2 + \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^3 \xi_i^3}{2 B_i^2} + \\
 & \frac{(-3 \gamma \mathcal{A}_i^2 + 7 \gamma B_i \mathcal{A}_i^2 - 5 \gamma B_i^2 \mathcal{A}_i^2 + \gamma B_i^3 \mathcal{A}_i^2) \beta_i \eta_i^3 \xi_i^3}{4 h^2 B_i^2} + \frac{y_i (-9 \gamma^2 \mathcal{A}_i^2 + 15 \gamma^2 B_i \mathcal{A}_i^2 - 7 \gamma^2 B_i^2 \mathcal{A}_i^2 + \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^4 \xi_i^3}{8 h B_i^4} + \frac{1}{8} \gamma^2 h^2 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 + \\
 & \frac{\mathbf{x}_i^3 (-3 \gamma^2 h \mathcal{A}_i^2 + \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^4}{4 B_i} + \frac{\mathbf{x}_i^2 (6 \gamma^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i \mathcal{A}_i^2 + \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^4}{4 B_i^2} + \frac{\mathbf{x}_i (-9 \gamma^2 \mathcal{A}_i^2 + 15 \gamma^2 B_i \mathcal{A}_i^2 - 7 \gamma^2 B_i^2 \mathcal{A}_i^2 + \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^3 \xi_i^4}{8 h B_i^2} + \\
 & \left. \frac{1}{32 h^2 B_i^4} (9 \gamma^2 \mathcal{A}_i^4 - 24 \gamma^2 B_i \mathcal{A}_i^4 + 22 \gamma^2 B_i^2 \mathcal{A}_i^4 - 8 \gamma^2 B_i^3 \mathcal{A}_i^4 + \gamma^2 B_i^4 \mathcal{A}_i^4) \eta_i^4 \xi_i^4 \right) \in^2 + \mathcal{O}[\epsilon]^3]
 \end{aligned}$$

$$\begin{aligned}
 a\Delta \rightarrow \mathbb{E}_{\{i\} \rightarrow \{j, k\}} & \left[\mathbf{a}_j \alpha_i + \mathbf{a}_k \alpha_i, \mathbf{x}_j \xi_i + \mathbf{x}_k \xi_i, \right. \\
 1 + & \left(-h \mathbf{a}_j \mathbf{x}_k \xi_i + \frac{1}{2} \gamma h \mathbf{x}_j \mathbf{x}_k \xi_i^2 \right) \in + \left(\frac{1}{2} h^2 \mathbf{a}_j^2 \mathbf{x}_k \xi_i + \frac{1}{4} \gamma^2 h^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \gamma h^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{2} h^2 \mathbf{a}_j^2 \mathbf{x}_k^2 \xi_i^2 + \right. \\
 & \left. \frac{1}{6} \gamma^2 h^2 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^2 h^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{1}{2} \gamma h^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 + \frac{1}{8} \gamma^2 h^2 \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 \right) \in^2 + \mathcal{O}[\epsilon]^3]
 \end{aligned}$$

$$\begin{aligned}
 b\Delta \rightarrow \mathbb{E}_{\{i\} \rightarrow \{j, k\}} & \left[\mathbf{b}_j \beta_i + \mathbf{b}_k \beta_i, \mathbf{B}_k y_j \eta_i + \mathbf{y}_k \eta_i, 1 + \frac{1}{2} \gamma h \mathbf{B}_k y_j y_k \eta_i^2 \in + \right. \\
 & \left. \left(\frac{1}{4} \gamma^2 h^2 \mathbf{B}_k y_j y_k \eta_i^2 + \frac{1}{6} \gamma^2 h^2 \mathbf{B}_k^2 y_j^2 y_k \eta_i^3 + \frac{1}{6} \gamma^2 h^2 \mathbf{B}_k y_j y_k^2 \eta_i^3 + \frac{1}{8} \gamma^2 h^2 \mathbf{B}_k^2 y_j^2 y_k^2 \eta_i^4 \right) \in^2 + \mathcal{O}[\epsilon]^3 \right]
 \end{aligned}$$

$$\begin{aligned}
d\Delta &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{j,k\}} \left[\mathbf{a}_j \alpha_i + \mathbf{a}_k \alpha_i + \mathbf{b}_j \beta_i + \mathbf{b}_k \beta_i, \right. \\
&\quad \mathbf{y}_j \eta_i + \mathbf{B}_j \mathbf{y}_k \eta_i + \mathbf{x}_j \xi_i + \mathbf{x}_k \xi_i, \mathbf{1} + \left(\frac{1}{2} \gamma \hbar \mathbf{B}_j \mathbf{y}_j \mathbf{y}_k \eta_i^2 - \hbar \mathbf{a}_j \mathbf{x}_k \xi_i + \frac{1}{2} \gamma \hbar \mathbf{x}_j \mathbf{x}_k \xi_i^2 \right) \epsilon + \\
&\quad \left(\frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_j \mathbf{y}_j \mathbf{y}_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_j \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_j^2 \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{B}_j^2 \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 + \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k \xi_i - \right. \\
&\quad \left. \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{B}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k^2 \xi_i^2 + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j \right. \\
&\quad \left. \mathbf{y}_k \eta_i^2 \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 \right) \epsilon^2 + \mathbf{O}[\epsilon^3] \\
\mathbf{C} &\rightarrow \mathbb{E}_{\{\} \rightarrow \{i\}} \left[\mathbf{0}, \mathbf{0}, \sqrt{\mathbf{B}_i} - \frac{1}{2} \left(\hbar \mathbf{a}_i \sqrt{\mathbf{B}_i} \right) \epsilon + \frac{1}{8} \hbar^2 \mathbf{a}_i^2 \sqrt{\mathbf{B}_i} \epsilon^2 + \mathbf{O}[\epsilon^3] \right] \\
\overline{\mathbf{C}} &\rightarrow \mathbb{E}_{\{\} \rightarrow \{i\}} \left[\mathbf{0}, \mathbf{0}, \frac{1}{\sqrt{\mathbf{B}_i}} + \frac{\hbar \mathbf{a}_i \epsilon}{2\sqrt{\mathbf{B}_i}} + \frac{\hbar^2 \mathbf{a}_i^2 \epsilon^2}{8\sqrt{\mathbf{B}_i}} + \mathbf{O}[\epsilon^3] \right] \\
\mathbf{Kink} &\rightarrow \mathbb{E}_{\{\} \rightarrow \{i\}} \left[\hbar \mathbf{a}_i \mathbf{b}_i, \hbar \mathbf{x}_i \mathbf{y}_i, \right. \\
&\quad \left. \frac{1}{\sqrt{\mathbf{B}_i}} + \left(\frac{\hbar \mathbf{a}_i}{2\sqrt{\mathbf{B}_i}} - \frac{\gamma \hbar^3 \mathbf{x}_i^2 \mathbf{y}_i^2}{4\sqrt{\mathbf{B}_i}} \right) \epsilon + \left(\frac{\hbar^2 \mathbf{a}_i^2}{8\sqrt{\mathbf{B}_i}} - \frac{\gamma \hbar^4 \mathbf{a}_i \mathbf{x}_i^2 \mathbf{y}_i^2}{8\sqrt{\mathbf{B}_i}} + \frac{\gamma^2 \hbar^5 \mathbf{x}_i^3 \mathbf{y}_i^3}{9\sqrt{\mathbf{B}_i}} + \frac{\gamma^2 \hbar^6 \mathbf{x}_i^4 \mathbf{y}_i^4}{32\sqrt{\mathbf{B}_i}} \right) \epsilon^2 + \mathbf{O}[\epsilon^3] \right] \\
\overline{\mathbf{Kink}} &\rightarrow \mathbb{E}_{\{\} \rightarrow \{i\}} \left[-\hbar \mathbf{a}_i \mathbf{b}_i, -\frac{\hbar \mathbf{x}_i \mathbf{y}_i}{\mathbf{B}_i}, \sqrt{\mathbf{B}_i} + \left(-\frac{1}{2} \hbar \mathbf{a}_i \sqrt{\mathbf{B}_i} - \frac{\hbar^2 \mathbf{a}_i \mathbf{x}_i \mathbf{y}_i}{\sqrt{\mathbf{B}_i}} - \frac{3\gamma \hbar^3 \mathbf{x}_i^2 \mathbf{y}_i^2}{4\mathbf{B}_i^{3/2}} \right) \epsilon + \right. \\
&\quad \left. \left(\frac{1}{8} \hbar^2 \mathbf{a}_i^2 \sqrt{\mathbf{B}_i} + \frac{\gamma^2 \hbar^4 \mathbf{x}_i^2 \mathbf{y}_i^2}{2\mathbf{B}_i^{3/2}} - \frac{9\gamma \hbar^4 \mathbf{a}_i \mathbf{x}_i^2 \mathbf{y}_i^2}{8\mathbf{B}_i^{3/2}} + \frac{\hbar^4 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathbf{y}_i^2}{2\mathbf{B}_i^{3/2}} - \frac{10\gamma^2 \hbar^5 \mathbf{x}_i^3 \mathbf{y}_i^3}{9\mathbf{B}_i^{5/2}} + \frac{3\gamma \hbar^5 \mathbf{a}_i \mathbf{x}_i^3 \mathbf{y}_i^3}{4\mathbf{B}_i^{5/2}} + \frac{9\gamma^2 \hbar^6 \mathbf{x}_i^4 \mathbf{y}_i^4}{32\mathbf{B}_i^{7/2}} \right) \epsilon^2 + \mathbf{O}[\epsilon^3] \right] \\
\mathbf{b2t} &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[\mathbf{a}_i \alpha_i - \frac{\mathbf{t}_i \beta_i}{\gamma}, \mathbf{y}_i \eta_i + \mathbf{x}_i \xi_i, \mathbf{1} + \frac{\mathbf{a}_i \beta_i \epsilon}{\gamma} + \frac{\mathbf{a}_i^2 \beta_i^2 \epsilon^2}{2\gamma^2} + \mathbf{O}[\epsilon^3] \right] \\
\mathbf{t2b} &\rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[\mathbf{a}_i \alpha_i - \gamma \mathbf{b}_i \tau_i, \mathbf{y}_i \eta_i + \mathbf{x}_i \xi_i, \mathbf{1} + \mathbf{a}_i \tau_i \epsilon + \frac{1}{2} \mathbf{a}_i^2 \tau_i^2 \epsilon^2 + \mathbf{O}[\epsilon^3] \right]
\end{aligned}$$

```

In[*]:= Print[degrule = Thread[{a, b, α, β, ξ, η, x, y, ħ, γ, ε, t, τ, T, B, A} →
  {1, 1, -1, -1, -1, -1, 1, 1, -2, 1, 1, 2, -2, 0, 0, 0}]];
atoms /. E_sp__[L_, Q_, P_] := (E[L, Q, P] ≡
  (E[L, Q, Normal@P] /. {v_i_ := s^v/.degrule v_i, (v : ħ | ε | γ) := s^v/.degrule v}))

{a → 1, b → 1, α → -1, β → -1, ξ → -1, η → -1, x → 1,
 y → 1, ħ → -2, γ → 1, ε → 1, t → 2, τ → -2, T → 0, B → 0, A → 0}

Out[*]:= {am → True, bm → True, dm → True, R → True, R̄ → True, P → True, aS → True,
  aS̄ → True, bS → True, bS̄ → True, dS → True, aΔ → True, bΔ → True, dΔ → True,
  C → True, C̄ → True, Kink → True, Kink̄ → True, b2t → True, t2b → True}

```

```

In[*]:= Print[degrule = Thread[{a, b, α, β, ξ, η, x, y, ħ, γ, ε, t, τ, T, B, A} →
  {0, 1, 0, -1, 0, -1, 0, 1, -1, 0, 1, 1, -1, 0, 0, 0}]];
atoms /. E_sp__[L_, Q_, P_] := (E[L, Q, P] ≡
  (E[L, Q, Normal@P] /. {v_i_ := s^v/.degrule v_i, (v : ħ | ε | γ) := s^v/.degrule v}))

{a → 0, b → 1, α → 0, β → -1, ξ → 0, η → -1, x → 0,
 y → 1, ħ → -1, γ → 0, ε → 1, t → 1, τ → -1, T → 0, B → 0, A → 0}

Out[*]:= {am → True, bm → True, dm → True, R → True, R̄ → True, P → True, aS → True,
  aS̄ → True, bS → True, bS̄ → True, dS → True, aΔ → True, bΔ → True, dΔ → True,
  C → True, C̄ → True, Kink → True, Kink̄ → True, b2t → True, t2b → True}

```

```
In[*]:= Print[degrule = Thread[{a, b, α, β, ξ, η, x, y, ħ, γ, ε, t, τ, T, B, A} →
    {1, 0, -1, 0, -1, 0, 1, 0, -1, 1, 0, 1, -1, 0, 0, 0}]];
atoms /. E_sp__[L_, Q_, P_] := (E[L, Q, P] ≡
    (E[L, Q, Normal@P] /. {v_i_ := s^v/.degrule v_i, (v : ħ | ε | γ) := s^v/.degrule v})))
```

```
{a → 1, b → 0, α → -1, β → 0, ξ → -1, η → 0, x → 1,
y → 0, ħ → -1, γ → 1, ε → 0, t → 1, τ → -1, T → 0, B → 0, A → 0}
```

```
Out[*]:= {am → True, bm → True, dm → True, R → True, R̄ → True, P → True, aS → True,
aS̄ → True, bS → True, bS̄ → True, dS → True, aΔ → True, bΔ → True, dΔ → True,
C → True, C̄ → True, Kink → True, Kink̄ → True, b2t → True, t2b → True}
```

```
In[*]:= Column[atoms /. E_sp__[L_, Q_, P_] := EE_sp[L, Q, CF@Log[P]]]
```

am → EE_{{i,j}→{k}} [a_k (α_i + α_j), x_k (e^{-γ α_j} ξ_i + ξ_j), 0]

bm → EE_{{i,j}→{k}} [b_k (β_i + β_j), y_k (η_i + η_j), -y_k β_i η_j ε + 1/2 y_k β_i² η_j ε² + 0[ε]³]

dm → EE_{{i,j}→{k}} [a_k α_i + a_k α_j + b_k β_i + b_k β_j, y_k η_i + $\frac{y_k \eta_j}{\mathcal{A}_i} + \frac{x_k \xi_i}{\mathcal{A}_j} + \frac{(1-B_k) \eta_j \xi_i}{\hbar} + x_k \xi_j$,
 $\left(-\frac{y_k \beta_i \eta_j}{\mathcal{A}_i} - \frac{x_k \beta_j \xi_i}{\mathcal{A}_j} + a_k B_k \eta_j \xi_i + \frac{\gamma \hbar x_k y_k \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} + \frac{(\gamma-3\gamma B_k) y_k \eta_j^2 \xi_i}{2 \mathcal{A}_i} + \frac{(\gamma-3\gamma B_k) x_k \eta_j \xi_i^2}{2 \mathcal{A}_j} + \frac{(\gamma-4\gamma B_k+3\gamma B_k^2) \eta_j^2 \xi_i^2}{4 \hbar} \right) \epsilon +$
 $\left(\frac{y_k \beta_i^2 \eta_j}{2 \mathcal{A}_i} + \frac{x_k \beta_j^2 \xi_i}{2 \mathcal{A}_j} - \frac{1}{2} \hbar a_k^2 B_k \eta_j \xi_i + \frac{\gamma^2 \hbar^2 x_k y_k \eta_j \xi_i}{2 \mathcal{A}_i \mathcal{A}_j} - \frac{\gamma \hbar x_k y_k \beta_i \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} - \frac{\gamma \hbar x_k y_k \beta_j \eta_j \xi_i}{\mathcal{A}_i \mathcal{A}_j} + \frac{3\gamma \hbar a_k B_k y_k \eta_j^2 \xi_i}{2 \mathcal{A}_i} + \right.$
 $\frac{(\gamma^2 \hbar-5\gamma^2 \hbar B_k) y_k \eta_j^2 \xi_i}{4 \mathcal{A}_i} + \frac{\gamma^2 \hbar^2 x_k y_k^2 \eta_j^2 \xi_i}{2 \mathcal{A}_i^2 \mathcal{A}_j} + \frac{(-\gamma+3\gamma B_k) y_k \beta_i \eta_j^2 \xi_i}{2 \mathcal{A}_i} + \frac{(\gamma^2 \hbar-7\gamma^2 \hbar B_k) y_k^2 \eta_j^2 \xi_i}{6 \mathcal{A}_i^2} + \frac{\gamma^2 \hbar^2 x_k^2 y_k \eta_j \xi_i^2}{2 \mathcal{A}_i \mathcal{A}_j^2} +$
 $\frac{3\gamma \hbar a_k B_k x_k \eta_j \xi_i^2}{2 \mathcal{A}_j} + \frac{(\gamma^2 \hbar-5\gamma^2 \hbar B_k) x_k \eta_j \xi_i^2}{4 \mathcal{A}_j} + \frac{(-\gamma+3\gamma B_k) x_k \beta_j \eta_j \xi_i^2}{2 \mathcal{A}_j} + \frac{1}{2} a_k (2\gamma B_k - 3\gamma B_k^2) \eta_j^2 \xi_i^2 +$
 $\frac{1}{8} (\gamma^2 - 6\gamma^2 B_k + 5\gamma^2 B_k^2) \eta_j^2 \xi_i^2 + \frac{(5\gamma^2 \hbar-21\gamma^2 \hbar B_k) x_k y_k \eta_j^2 \xi_i^2}{4 \mathcal{A}_i \mathcal{A}_j} + \frac{(5\gamma^2-34\gamma^2 B_k+41\gamma^2 B_k^2) y_k \eta_j^2 \xi_i^2}{12 \mathcal{A}_i} +$
 $\left. \frac{(\gamma^2 \hbar-7\gamma^2 \hbar B_k) x_k^2 \eta_j \xi_i^3}{6 \mathcal{A}_i^2} + \frac{(5\gamma^2-34\gamma^2 B_k+41\gamma^2 B_k^2) x_k \eta_j^2 \xi_i^3}{12 \mathcal{A}_j} + \frac{(5\gamma^2-39\gamma^2 B_k+75\gamma^2 B_k^2-41\gamma^2 B_k^3) \eta_j^3 \xi_i^3}{36 \hbar} \right) \epsilon^2 + 0[\epsilon]^3$

R → EE_{{i}→{i,j}} [ħ a_j b_i, ħ x_j y_i, -1/4 (γ ħ³ x_j² y_i²) ε + 1/9 γ² ħ⁵ x_j³ y_i³ ε² + 0[ε]³]

R̄ → EE_{{i}→{i,j}} [-ħ a_j b_i, - $\frac{\hbar x_j y_i}{B_i}$,
 $\left(-\frac{\hbar^2 a_j x_j y_i}{B_i} - \frac{3\gamma \hbar^3 x_j^2 y_i^2}{4 B_i^2} \right) \epsilon + \left(-\frac{\hbar^3 a_j^2 x_j y_i}{2 B_i} + \frac{\gamma^2 \hbar^4 x_j^2 y_i^2}{2 B_i^2} - \frac{3\gamma \hbar^4 a_j x_j^2 y_i^2}{2 B_i^2} - \frac{10\gamma^2 \hbar^5 x_j^3 y_i^3}{9 B_i^2} \right) \epsilon^2 + 0[\epsilon]^3$

P → EE_{{i,j}→{i}} [$\frac{\alpha_j \beta_i}{\hbar}$, $\frac{\eta_i \xi_j}{\hbar}$, $\frac{\gamma \eta_j^2 \xi_j^2 \epsilon}{4 \hbar} + \left(\frac{1}{8} \gamma^2 \eta_i^2 \xi_j^2 + \frac{5\gamma^2 \eta_i^3 \xi_j^3}{36 \hbar} \right) \epsilon^2 + 0[\epsilon]^3$]

aS → EE_{{i}→{i}} [-a_i α_i, -x_i A_i ξ_i, (-ħ a_i x_i A_i ξ_i - 1/2 γ ħ x_i² A_i² ξ_i²) ε +
 $\left(-\frac{1}{2} \hbar^2 a_i^2 x_i A_i \xi_i + \frac{1}{4} \gamma^2 \hbar^2 x_i^2 A_i^2 \xi_i^2 - \gamma \hbar^2 a_i x_i^2 A_i^2 \xi_i^2 - \frac{1}{2} \gamma^2 \hbar^2 x_i^3 A_i^3 \xi_i^3 \right) \epsilon^2 + 0[\epsilon]^3$

aS̄ → EE_{{i}→{i}} [-a_i α_i, -x_i A_i ξ_i,
 $\left(\gamma \hbar x_i A_i \xi_i - \hbar a_i x_i A_i \xi_i - \frac{1}{2} \gamma \hbar x_i^2 A_i^2 \xi_i^2 \right) \epsilon + \left(-\frac{1}{2} \gamma^2 \hbar^2 x_i A_i \xi_i + \gamma \hbar^2 a_i x_i A_i \xi_i - \right.$
 $\left. \frac{1}{2} \hbar^2 a_i^2 x_i A_i \xi_i + \frac{5}{4} \gamma^2 \hbar^2 x_i^2 A_i^2 \xi_i^2 - \gamma \hbar^2 a_i x_i^2 A_i^2 \xi_i^2 - \frac{1}{2} \gamma^2 \hbar^2 x_i^3 A_i^3 \xi_i^3 \right) \epsilon^2 + 0[\epsilon]^3$

bS →

EE_{{i}→{i}} [-b_i β_i, - $\frac{y_i \eta_i}{B_i}$, $\left(-\frac{y_i \beta_i \eta_i}{B_i} - \frac{\gamma \hbar y_i^2 \eta_i^2}{2 B_i^2} \right) \epsilon + \left(-\frac{y_i \beta_i^2 \eta_i}{2 B_i} + \frac{\gamma^2 \hbar^2 y_i^2 \eta_i^2}{4 B_i^2} - \frac{\gamma \hbar y_i^2 \beta_i \eta_i}{B_i^2} - \frac{\gamma^2 \hbar^2 y_i^3 \eta_i^3}{2 B_i^2} \right) \epsilon^2 + 0[\epsilon]^3$

bS̄ → EE_{{i}→{i}} [-b_i β_i, - $\frac{y_i \eta_i}{B_i}$, $\left(\frac{\gamma \hbar y_i \eta_i}{B_i} - \frac{y_i \beta_i \eta_i}{B_i} - \frac{\gamma \hbar y_i^2 \eta_i^2}{2 B_i^2} \right) \epsilon +$
 $\left(-\frac{\gamma^2 \hbar^2 y_i \eta_i}{2 B_i} + \frac{\gamma \hbar y_i \beta_i \eta_i}{B_i} - \frac{y_i \beta_i^2 \eta_i}{2 B_i} + \frac{5\gamma^2 \hbar^2 y_i^2 \eta_i^2}{4 B_i^2} - \frac{\gamma \hbar y_i^2 \beta_i \eta_i}{B_i^2} - \frac{\gamma^2 \hbar^2 y_i^3 \eta_i^3}{2 B_i^2} \right) \epsilon^2 + 0[\epsilon]^3$

$$\begin{aligned}
 dS \rightarrow EE_{\{i\} \rightarrow \{i\}} & \left[-\mathbf{a}_i \alpha_i - \mathbf{b}_i \beta_i, -\frac{y_i \mathcal{A}_i \eta_i}{B_i} - \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \eta_i \xi_i}{\hbar B_i}, \right. \\
 & \left(\frac{\gamma \hbar y_i \mathcal{A}_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{\gamma \hbar y_i^2 \mathcal{A}_i^2 \eta_i^2}{2 B_i^2} - \hbar \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i + \frac{\mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} - \right. \\
 & \left. \frac{\gamma \hbar \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{\hbar B_i} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{2 B_i^2} - \right. \\
 & \left. \frac{1}{2} \gamma \hbar \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 \hbar B_i^2} \right) \in + \\
 & \left(-\frac{\gamma^2 \hbar^2 y_i \mathcal{A}_i \eta_i}{2 B_i} + \frac{\gamma \hbar y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i^2 \eta_i}{2 B_i} + \frac{5 \gamma^2 \hbar^2 y_i^2 \mathcal{A}_i^2 \eta_i^2}{4 B_i^2} - \frac{\gamma \hbar y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{B_i^2} - \frac{\gamma^2 \hbar^2 y_i^3 \mathcal{A}_i^3 \eta_i^3}{2 B_i^3} - \frac{1}{2} \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i - \right. \\
 & \hbar \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i - \frac{1}{2} \mathbf{x}_i \mathcal{A}_i \beta_i^2 \xi_i - \frac{\gamma \hbar \mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} + \frac{\hbar \mathbf{a}_i^2 \mathcal{A}_i \eta_i \xi_i}{2 B_i} + \frac{3 \gamma^2 \hbar^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{2 B_i} - \frac{\gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \\
 & \frac{(\gamma^2 \hbar \mathcal{A}_i - \gamma^2 \hbar B_i \mathcal{A}_i) \eta_i \xi_i}{2 B_i} + \frac{\mathbf{a}_i \mathcal{A}_i \beta_i \eta_i \xi_i}{B_i} - \frac{2 \gamma \hbar \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{2 \hbar B_i} + \\
 & \frac{3 \gamma \hbar \mathbf{a}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} - \frac{3 \gamma^2 \hbar^2 \mathbf{x}_i y_i^2 \mathcal{A}_i^3 \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (-17 \gamma^2 \hbar \mathcal{A}_i^2 + 5 \gamma^2 \hbar B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{4 B_i^2} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i}{B_i^2} + \\
 & \frac{y_i^2 (8 \gamma^2 \hbar \mathcal{A}_i^2 - 2 \gamma^2 \hbar B_i \mathcal{A}_i^2) \eta_i^3 \xi_i}{3 B_i^3} + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 - \frac{3 \gamma^2 \hbar^2 \mathbf{x}_i^2 y_i \mathcal{A}_i^3 \eta_i \xi_i^2}{2 B_i} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i (6 \gamma \hbar \mathcal{A}_i^2 - \gamma \hbar B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{x}_i (-11 \gamma^2 \hbar \mathcal{A}_i^2 + 3 \gamma^2 \hbar B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{4 B_i} + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{a}_i (-3 \gamma \mathcal{A}_i^2 + 2 \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \\
 & \frac{(17 \gamma^2 \mathcal{A}_i^2 - 22 \gamma^2 B_i \mathcal{A}_i^2 + 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{8 B_i^2} + \frac{\mathbf{x}_i y_i (21 \gamma^2 \hbar \mathcal{A}_i^2 - 5 \gamma^2 \hbar B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{2 \hbar B_i^2} + \\
 & \frac{y_i (-41 \gamma^2 \mathcal{A}_i^2 + 34 \gamma^2 B_i \mathcal{A}_i^2 - 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{12 B_i^3} - \frac{1}{2} \gamma^2 \hbar^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{\mathbf{x}_i^2 (8 \gamma^2 \hbar \mathcal{A}_i^2 - 2 \gamma^2 \hbar B_i \mathcal{A}_i^2) \eta_i \xi_i^3}{3 B_i} + \\
 & \left. \frac{\mathbf{x}_i (-41 \gamma^2 \mathcal{A}_i^2 + 34 \gamma^2 B_i \mathcal{A}_i^2 - 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{12 B_i^2} + \frac{(41 \gamma^2 \mathcal{A}_i^2 - 75 \gamma^2 B_i \mathcal{A}_i^2 + 39 \gamma^2 B_i^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^3 \xi_i^3}{36 \hbar B_i^3} \right) \in^2 + \mathcal{O}[\epsilon]^3
 \end{aligned}$$

$$\begin{aligned}
 a\Delta \rightarrow EE_{\{i\} \rightarrow \{j,k\}} & \left[\mathbf{a}_j \alpha_i + \mathbf{a}_k \alpha_i, \mathbf{x}_j \xi_i + \mathbf{x}_k \xi_i, \left(-\hbar \mathbf{a}_j \mathbf{x}_k \xi_i + \frac{1}{2} \gamma \hbar \mathbf{x}_j \mathbf{x}_k \xi_i^2 \right) \in + \right. \\
 & \left. \left(\frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k \xi_i + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 \right) \in^2 + \mathcal{O}[\epsilon]^3 \right]
 \end{aligned}$$

$$\begin{aligned}
 b\Delta \rightarrow EE_{\{i\} \rightarrow \{j,k\}} & \left[\mathbf{b}_j \beta_i + \mathbf{b}_k \beta_i, \mathbf{B}_k y_j \eta_i + \mathbf{y}_k \eta_i, \right. \\
 & \left. \frac{1}{2} \gamma \hbar \mathbf{B}_k y_j y_k \eta_i^2 \in + \left(\frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_k y_j y_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_k^2 y_j^2 y_k \eta_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_k y_j y_k^2 \eta_i^3 \right) \in^2 + \mathcal{O}[\epsilon]^3 \right]
 \end{aligned}$$

$$\begin{aligned}
 d\Delta \rightarrow EE_{\{i\} \rightarrow \{j,k\}} & \left[\mathbf{a}_j \alpha_i + \mathbf{a}_k \alpha_i + \mathbf{b}_j \beta_i + \mathbf{b}_k \beta_i, \right. \\
 & \left. y_j \eta_i + \mathbf{B}_j y_k \eta_i + \mathbf{x}_j \xi_i + \mathbf{x}_k \xi_i, \left(\frac{1}{2} \gamma \hbar \mathbf{B}_j y_j y_k \eta_i^2 - \hbar \mathbf{a}_j \mathbf{x}_k \xi_i + \frac{1}{2} \gamma \hbar \mathbf{x}_j \mathbf{x}_k \xi_i^2 \right) \in + \right. \\
 & \left. \left(\frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_j y_j y_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_j y_j^2 y_k \eta_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_j^2 y_j y_k^2 \eta_i^3 + \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k \xi_i + \right. \right. \\
 & \left. \left. \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 \right) \in^2 + \mathcal{O}[\epsilon]^3 \right]
 \end{aligned}$$

$$C \rightarrow EE_{\{\} \rightarrow \{i\}} \left[\mathbf{0}, \mathbf{0}, \frac{\text{Log}[B_i]}{2} - \frac{1}{2} (\hbar \mathbf{a}_i) \in + \mathcal{O}[\epsilon]^3 \right]$$

$$\bar{C} \rightarrow EE_{\{\} \rightarrow \{i\}} \left[\mathbf{0}, \mathbf{0}, -\frac{\text{Log}[B_i]}{2} + \frac{1}{2} \hbar \mathbf{a}_i \in + \mathcal{O}[\epsilon]^3 \right]$$

$$\text{Kink} \rightarrow EE_{\{\} \rightarrow \{i\}} \left[\hbar \mathbf{a}_i \mathbf{b}_i, \hbar \mathbf{x}_i y_i, -\frac{\text{Log}[B_i]}{2} + \left(\frac{\hbar \mathbf{a}_i}{2} - \frac{1}{4} \gamma \hbar^3 \mathbf{x}_i^2 y_i^2 \right) \in + \frac{1}{9} \gamma^2 \hbar^5 \mathbf{x}_i^3 y_i^3 \in^2 + \mathcal{O}[\epsilon]^3 \right]$$

$$\begin{aligned}
 \overline{\text{Kink}} \rightarrow EE_{\{\} \rightarrow \{i\}} & \left[-\hbar \mathbf{a}_i \mathbf{b}_i, -\frac{\hbar \mathbf{x}_i y_i}{B_i}, \right. \\
 & \left. \frac{\text{Log}[B_i]}{2} + \left(-\frac{\hbar \mathbf{a}_i}{2} - \frac{\hbar^2 \mathbf{a}_i \mathbf{x}_i y_i}{B_i} - \frac{3 \gamma \hbar^3 \mathbf{x}_i^2 y_i^2}{4 B_i^2} \right) \in + \left(-\frac{\hbar^3 \mathbf{a}_i^2 \mathbf{x}_i y_i}{2 B_i} + \frac{\gamma^2 \hbar^4 \mathbf{x}_i^2 y_i^2}{2 B_i^2} - \frac{3 \gamma \hbar^4 \mathbf{a}_i \mathbf{x}_i^2 y_i^2}{2 B_i^2} - \frac{10 \gamma^2 \hbar^5 \mathbf{x}_i^3 y_i^3}{9 B_i^3} \right) \in^2 + \mathcal{O}[\epsilon]^3 \right]
 \end{aligned}$$

$$b2t \rightarrow EE_{\{i\} \rightarrow \{i\}} \left[\mathbf{a}_i \alpha_i - \frac{\tau_i \beta_i}{\gamma}, y_i \eta_i + \mathbf{x}_i \xi_i, \frac{\mathbf{a}_i \beta_i \in}{\gamma} + \mathcal{O}[\epsilon]^3 \right]$$

$$t2b \rightarrow EE_{\{i\} \rightarrow \{i\}} \left[\mathbf{a}_i \alpha_i - \gamma \mathbf{b}_i \tau_i, y_i \eta_i + \mathbf{x}_i \xi_i, \mathbf{a}_i \tau_i \in + \mathcal{O}[\epsilon]^3 \right]$$

In[*]= **degs = atoms / . IE_{sp} [L_, Q_, P_] :=**

Exponent [Normal@CF@Log@P / . { (v : x | y | xi | eta)_i := lambda v_i, (v : a | beta)_i := lambda^2 v_i }, lambda]

Out[*]= { am -> -inf, bm -> 4, dm -> 4, R -> 3, R-bar -> 3, P -> 3, aS -> 6, aS-bar -> 6, bS -> 6, bS-bar -> 6, dS -> 6, aD -> 3, bD -> 3, dD -> 3, C -> 2, C-bar -> 2, Kink -> 6, Kink-bar -> 6, b2t -> 4, t2b -> 2 }

In[*n*]:= **Last** /@ **degs**

Out[*n*]= { $-\infty$, 4, 4, 3, 3, 3, 6, 6, 6, 6, 6, 3, 3, 3, 2, 2, 6, 6, 4, 2}

In[*n*]:= **Max**[**Last** /@ **degs**]

Out[*n*]= 6